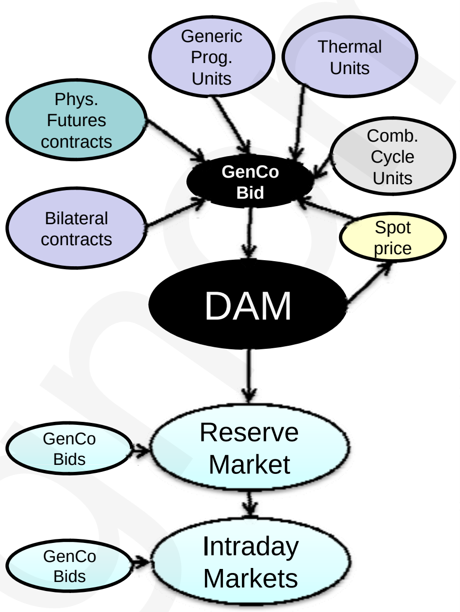
Proyecto MICINN DPI2008-02153

- Problema de optimización de la oferta al mercado diario.
- Modelos estocásticos de optimización de la oferta desarrollados hasta ahora.
- Trabajos pendientes.

(http://gnom.upc.edu/projects/energy/dpi2008-02153)

Day-Ahead Market (DAM) bid optimization



- F. J. Heredia, M. J. Rider, C. Corchero, "Optimal Bidding Strategies for Thermal and Generic Programming Units in the Day-ahead Electricity Market", *IEEE Transactions on Power Systems,* 2010.
- F. J. Heredia, M. J. Rider, C. Corchero,"Optimal Bidding Strategies for Thermal and Combined Cycle Units in the Day-ahead Electricity Market with Bilateral Contracts", *Proceedings of the 2009 Power Engineering Society General Meeting*, Calgary, Canada, 2009.
- C. Corchero, F. J. Heredia, "A Stochastic Programming Model for the Thermal Optimal Day-Ahead Bid Problem with Physical Futures Contracts", *Submitted to Computers and Operations Research.* 2010.
- C. Corchero, F.J. Heredia, " Optimal Day-Ahead Bidding in the MIBEL's Multimarket Energy Production System ", *Proceedings of the 7th Conference on European Energy Market EEM10*, Madrid, 2010.
- M. P. Muñoz, C. Corchero, F. J. Heredia, "Improving electricity market price scenarios by means of forecasting factor models". *Invited paper to International Statistical Review,* 2011.

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DAM bid stochastic optimization models

The models developped find the optimal value of:

- The DAM bid for all units (thermal, CC, GPU).
- The scheduled energy to each future and bilateral contract.
- The unit committment and CC unit's operation.

… by solving the following stochastic programming problem:

Max Expected profit from the market

Subject to:

Future and Bilateral contracts coverage

Day-ahead, reserve and intraday market rules

Minimum up/down time and CC unit's operation

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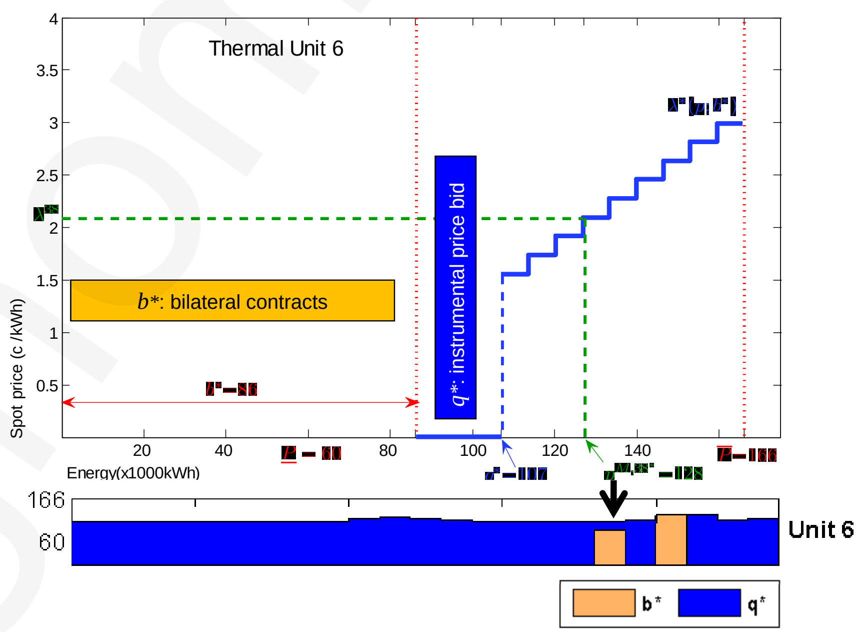
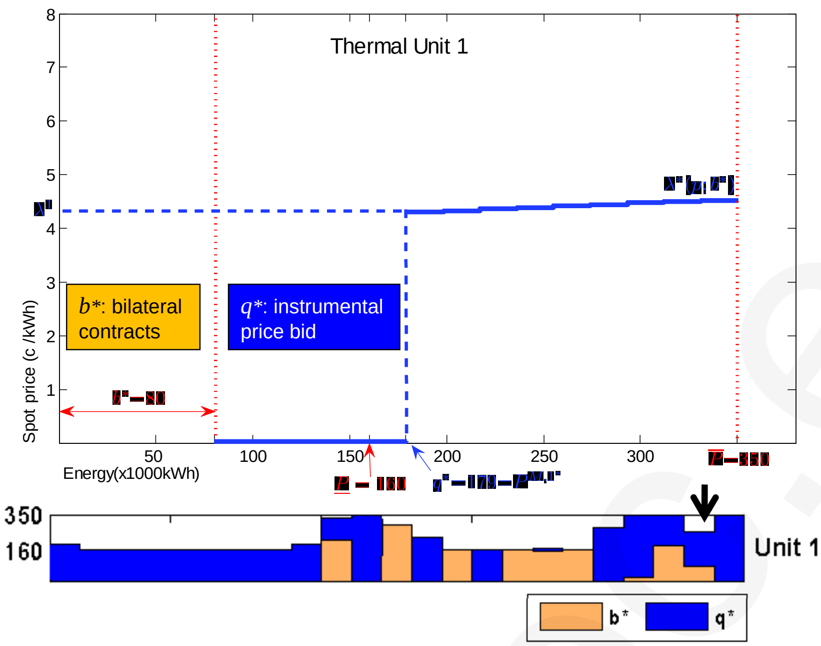
Thermal unit's optimal day-ahead bid

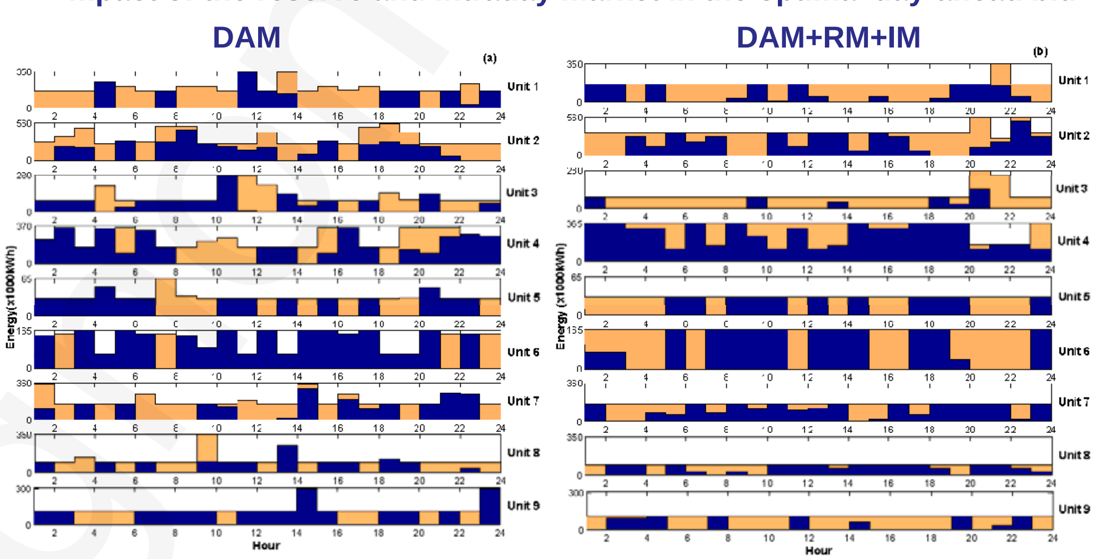


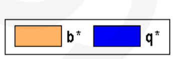
Thermal unit's optimal day-ahead bid



Thermal unit's optimal day-ahead bid

Impact of the reserve and intraday market in the optimal day-ahead bid





+8% profit with DAM+RM+IM

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Curva de oferta real CT Anllares

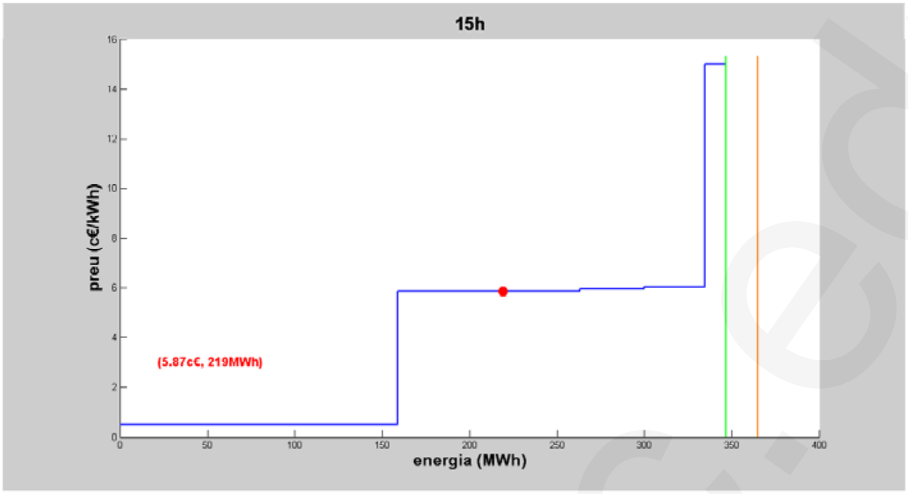

Fig 2.12: Corba d'oferta de la unitat ALL1 el dia 26 de maig de 2008 a les 15 hores

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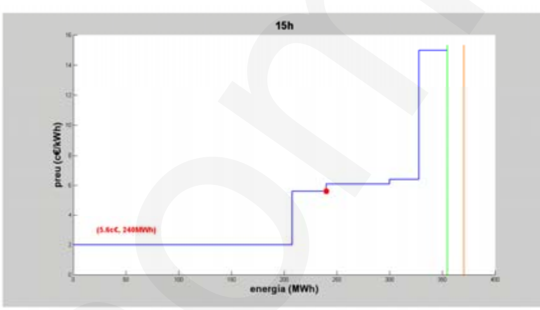
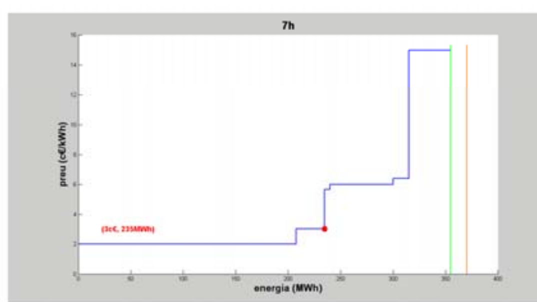
Curva de oferta real CT La Robla 2



2.17: Corba d'oferta de la unitat ROB2 durant el dia 8 de maig de 2008 a les 15 hores



2.21: Corba d'oferta de la unitat ROB2 durant el dia 12 de maig de 2008 a les 7 hores

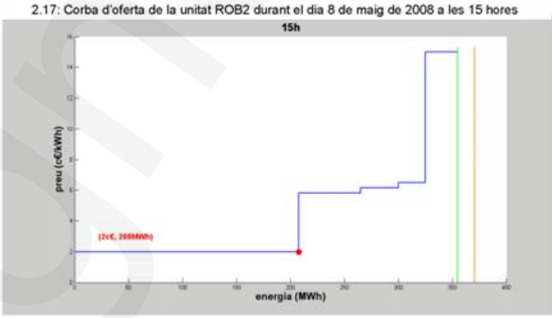
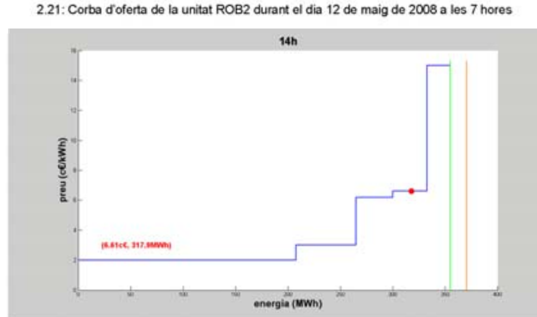


Fig. 2.25: Corba d'oferta de la unitat ROB2 durant el dia 19 de maig de 2008 a les 15 hores



2.29: Corba d'oferta de la unitat ROB2 el dia 27 de maig de 2008 a les 14 hores

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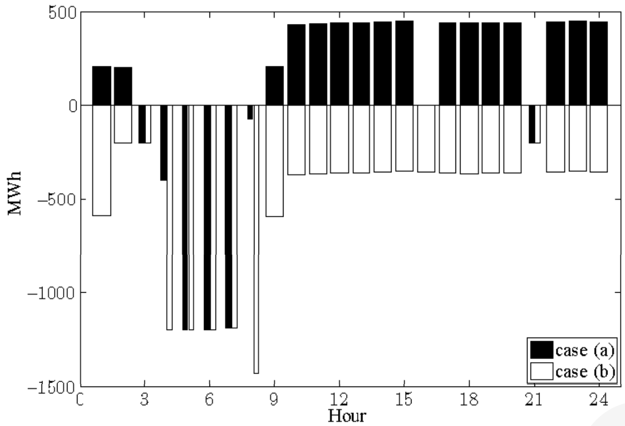
GPU and thermal unit's optimal day-ahead bid

Impact of the Generic Programming Unit (GPU):

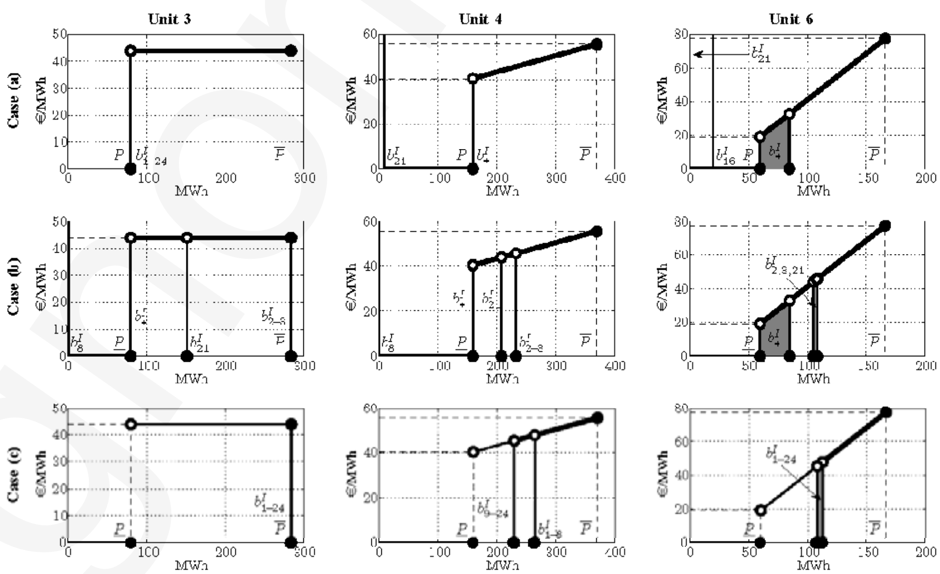
- **Case (a) :** Genco with GPU and VPP capacity (+47% profit)
- **Case (b) :** GenCo with GPU (+10% profit)
- **Case (c) :** Genco without GPU



Sold (>0) and bought (<0) optimal bid of the GPU for cases (a) and (b)

GPU and thermal unit's optimal day-ahead bid

Optimal bid of the thermal units:



+GPU+VPP

+GPU

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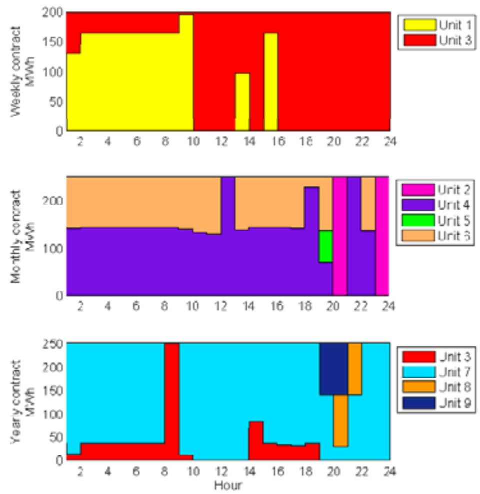
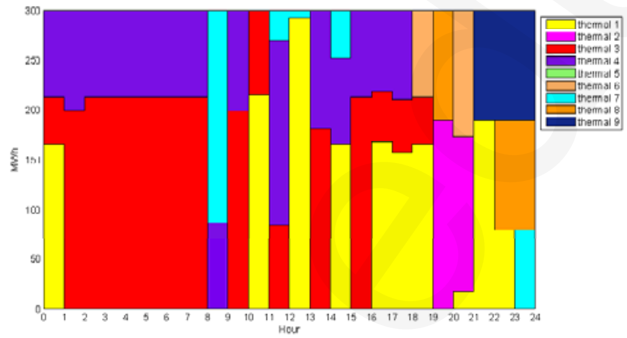
Scheduled energy to each future and bilateral contract

FC settlement:



BC settlement:



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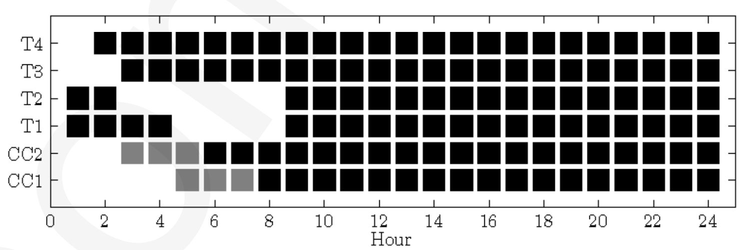
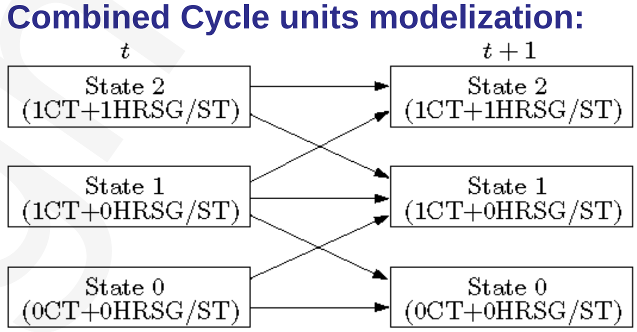
Unit committment and CC’s operation

Some results:



Combined Cycle units modelization:



- **CT :** combustion turbine.
- **HRSG/ST :** heat recovery steam generator/steam turbine

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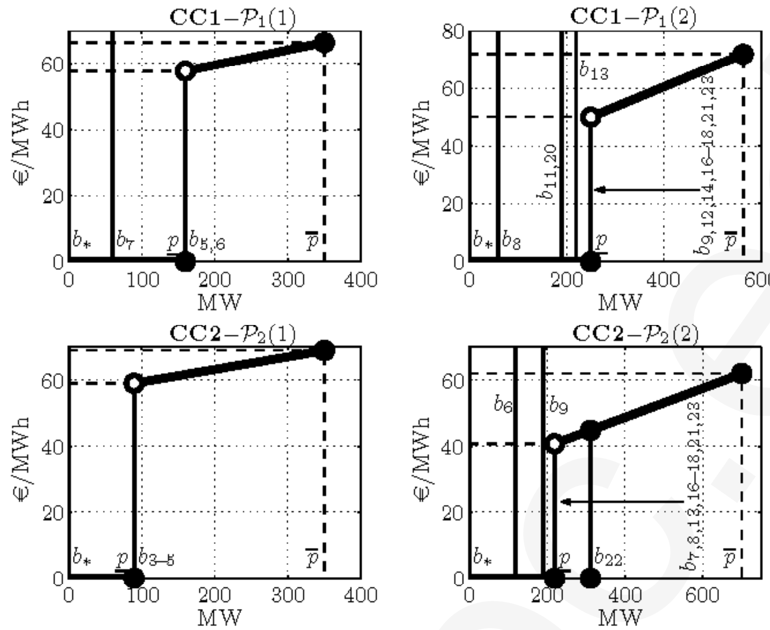
Combined Cycle unit's optimal day-ahead bid



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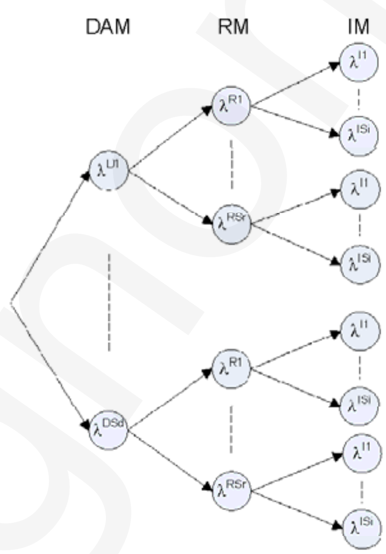
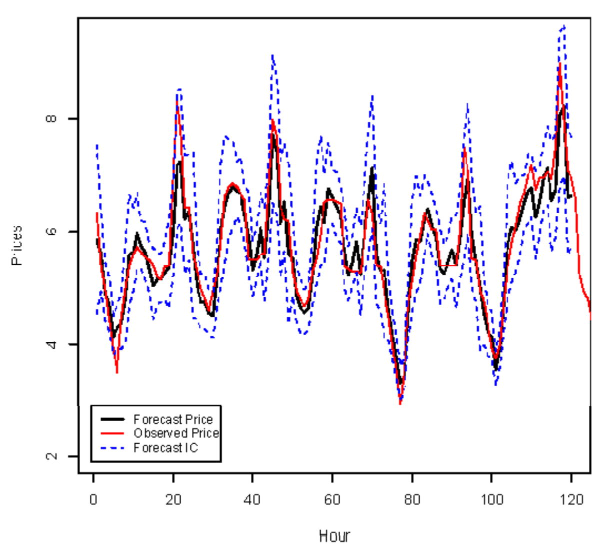
Stochastic programming

Scenario tree:



(1) Market price forecasting (TSFA):



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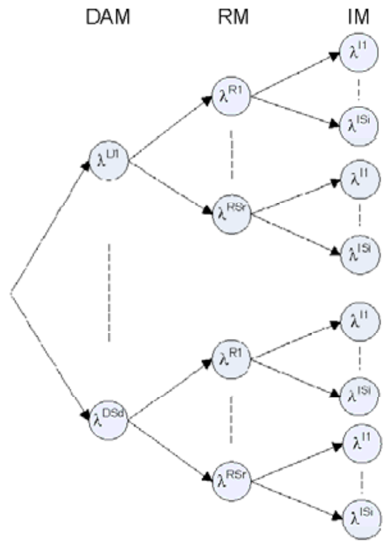
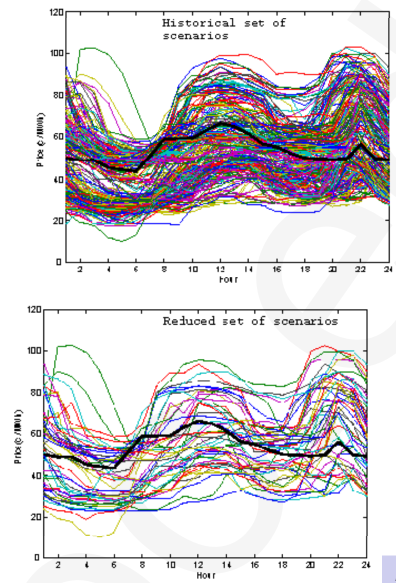
Stochastic programming

Scenario tree:



(2) Scenario generation and reduction:



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Expected profit from the DAM+RM+IM

$$E_{\lambda^{D,R,I}}\left[B(g,p,m,r,u,c^u,c^d)\right] = \sum_{t\in T}\left[\sum_{j\in F}(\lambda_j^F - \overline{\lambda}_i^D)L_j^F + \sum_{j\in B}\lambda_j^B L_j^B\right]$$

FC constant incomes/losses — **BC constant incomes**

$$+\sum_{t\in T}\sum_{i\in I}\sum_{s\in S} P^s\left[\lambda_t^{D,s}p_{ti}^s + \lambda_t^{R,s}\varrho_i r_{ti}^s + \lambda_t^{I,s}m_{ti}^s - (c_i^l g_{ti}^s + c_i^q (g_{it}^s)^2)\right]$$

Probability scenario "s" — **DAM incomes** — **RM incomes** — **IM incomes** — **Quadratic generation costs**

$$-\sum_{t\in T}\sum_{i\in I}[c_{ti}^u + c_{ti}^d + c_i^b u_{ti}]$$

Start-up /shut down costs

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Futures and bilateral contract coverage

MWh unit "i" to futures contract "j", period "t"

Total MWh futures contract "j"

$$\sum_{i \in I_j} f_{tij} = L_j^F \qquad \forall j \in F,\ \forall t \in T$$

MWh unit "i" to bilateral contracts pool, period "t"

Total MWh bilateral contracts pool

$$\sum_{i \in I} b_{ti} = \sum_{j \in B} L_j^B \qquad \forall t \in T$$

$$f_{tij} \geq 0 \qquad \forall j \in F,\ \forall i \in I,\ \forall t \in T$$

Unit committment {0,1} state of unit "i" period "t"

$$0 \leq b_{ti} \leq \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T$$

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Market modelization (1/3)

Instrumental DAM price bid: for each unit "i", period "t" and scenario "s"

$$\sum_{j | i \in I_j} f_{tij} \leq q_{ti} \leq p_{ti}^s$$

Total energy allocated to FC

Instrumental price DAM bid

DAM matched energy

PDBF result limits: for each unit "i", period "t" and scenario "s"

PDBF result scenario "s" $b_{ti} + p_{ti}^s \leq \overline{P}_i u_{ti}$ **Max. generation**

Min PDBF result. $b_{ti} + q_{ti} \geq \underline{P}_i u_{ti}$ **Min. generation**

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Market modelization (2/3)

Total generation: for each unit "i", period "t" and scenario "s"

$$g_{ti}^{s} = b_{ti} + p_{ti}^{s} + m_{ti}^{s}$$

Total generation — Energy to BC pool — DAM matched energy — IM matched energy

Limits to the total generation:

$$\underline{P}_i u_{ti} + \rho_i r_{it}^{s} \le g_{ti}^{s} \le \overline{P}_i u_{ti} - \rho_i r_{it}^{s}$$

RM bid variable indicator {0,1}

RM bid energy

$$r_{it}^{s} \le u_{ti}$$

Unit committment {0,1} state of unit "i" period "t"

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Market modelization (3/3)

RM participation: for each unit "i", period "t" and scenario "s"

$$g_{ti}^{s} - g_{(t-1),i}^{s} \in \left[-\overline{P}_i (1 - r_{ti}^{s}), \overline{P}_i (1 - r_{ti}^{s}) \right]$$

$$\left[-\overline{P}_i , \overline{P}_i \right] \quad \text{if } r_{ti}^{s} = 0$$

$$\left[0, 0 \right] \quad \text{if } r_{ti}^{s} = 1$$

Generation change between period "t" and "t-1"

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Medium-term generation planning optimization in liberalized electricity markets

N. Nabona and L. Marí
narcis.nabona@upc.edu & laura.mari@upc.edu
GNOM Group on Numerical Optimization and Modeling
Dept. Statistics and Operations Research
Universitat Politècnica de Catalunya, 08034 Barcelona

14th December 2010

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- **Medium-Term Mixed-System Electricity Markets**

Introduction

Medium term power planning

- The most reasonable way of evaluating the impact of renewable energies is through medium term planning.
- Medium term planning can also be used to find the equilibrium solution in electricity markets (through the Nikaido-Isoda algorithm of successive optimizations).
- The increase of risk of profit loss due to the use of renewables can be also evaluated.

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Generation units in medium-term planning

We should first distinguish between the specific generation company (SGC), of which we know its generation units detail, and the rest of participants (RoP) in the market, of which we know their generation units with less detail. The generation units to be considered are:

- all thermal units of the SGC whose production is to participate in the auction process,
- it would be good to consider the reservoir systems of hydro production of the SGC with full detail, but it is usual to model hydrogeneration of the SGC as one or several equivalent simplified single-reservoir systems with or without run-of-the-river,
- the thermal units of the RoP, either as single or as *merged* pseudo-units of similar characteristics (e.g., all available nuclear units of the competitor companies could be merged into a single nuclear pseudo-unit),
- the hydro-systems of the RoP considered as one or more single-reservoir schemes.
- big cascaded reservoirs can be taken into account with a detailed hydro model using extra variables.

Generation units in medium-term planning

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In medium-term planning the relevant parameters of a *thermal* unit are:

⋆ *power capacity*: c_j for the $j^{\underline{\text{th}}}$ unit (MW)

⋆ *outage probability*: q_j for the $j^{\underline{\text{th}}}$

⋆ *linear generation cost*: $\widetilde{f}_j$ for the $j^{\underline{\text{th}}}$ unit (€/MWh)

Let us denote by M the set of units merged into one given pseudo-unit, and let r be the index of one of the composing units. The parameters of the pseudo-unit can be calculated as:

- *maximum power capacity* $c_M = \sum_{r\in M} c_r$
- *linear generation cost* $f_M = \left(\sum_{r\in M} c_r\, f_r\right) / \sum_{r\in M} c_r$
- *outage probability* $q_M = \left(\sum_{r\in M} c_r\, q_r\right) / \sum_{r\in M} c_r$.

Natural water inflows in reservoirs (genuine ones or simplifications) are stochastic in medium-term planning, and scenarios should be employed.

The convolution method to match the load 1

The loading of thermal units to match an LDC was first formulated by Balériaux, Jamoulle and Linard de Guertechin in 1967. Let:

c_j : maximum power capacity in MW of unit j
q_j : outage probability of unit j
$1 - q_j$: in service probability of unit j
U_j : set of unit indices $1, 2, \ldots, j$
$S_{U_{j-1}}(z)$: load-survival function of unmatched load after loading units $1, 2, ..., j-1$ (z: load in MW)
$S_{U_j}(z)$: load-survival function of unmatched load after loading units $1, 2, ..., j-1, j$

the convolution computes $S_{U_j}(z)$ from $S_{U_{j-1}}(z)$ as:

$$S_{U_j}(z) = q_j\, S_{U_{j-1}}(z) + (1 - q_j)\, S_{U_{j-1}}(z + c_j)$$

Recalling that energy$= T \cdot p$, the energy generated by unit j is:

$$x_j = (1 - q_j)\, T \int_0^{c_j} S_{U_{j-1}}(z)\, dz \ .$$

Other associated concepts are:

⋆ *merit order*: units are loaded ordered according to their cost

⋆ *loading order*: units will have load allocated to them in a given order (due to active non-load-matching constraints).

The convolution method to match the load 3

Starting with $S_\emptyset(z)$ and convolving successively the units $1, 2, \ldots$ we will find the distribution of unsupplied load after *loading* these units. Given a set of units whose indices $1, 2, \ldots, n_u$ are the elements of the set of indices Ω, the unsupplied load after loading all the units in Ω will have a load-survival function $S_\Omega(z)$:

$$S_\Omega(z) = S_\emptyset(z) \prod_{m \in \Omega} q_m +$$
$$\sum_{U \subseteq \Omega} (S_\emptyset(z + \sum_{i \in U} c_i)(1 - q_i) \prod_{i \in U} (1 - q_i) \prod_{i \in U} q_i)$$

We can thus say that $S_\Omega(z)$ (of unsupplied load) is the same no matter in which order the units in Ω have been loaded. The unsupplied energy (external energy to be acquired) $w(\Omega)$ is:

$$w(\Omega) = T \int_0^{\widehat{p}} S_\Omega(z)\, dz$$

The unsupplied load after having loaded the units in subset $U \in \Omega$ is:

$$w(U) = T \int_0^{\widehat{p}} S_U(z)\, dz \, .$$

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Medium-Term Power Planning Optimization

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The multi-period Bloom and Gallant formulation 1

The Bloom and Gallant linear optimization model extended to n_i periods, and with inequality non-LMCs and no extra variables y is:

$$\begin{array}{ll}
\underset{x_j^i}{\text{minimize}} & \displaystyle\sum_{i=1}^{n_i}\sum_{j=0}^{n_u} \widetilde{f}_j x_j^i \\
\text{subject to:} & \displaystyle\sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i=1,\ldots,n_i \\
& \displaystyle\sum_{j=0}^{n_u} x_j^i = \widehat{e}^i \quad i=1,\ldots,n_i \\
& A^i x^i \ge r^i \quad i=1,\ldots,n_i \\
& \displaystyle\sum_i A^{0i} x^i \ge r^0 \\
& x_j^i \ge \underline{0} \qquad j=0,1,\ldots,n_u^i \quad i=1,\ldots,n_i
\end{array}$$

where supraindex ^{i} means relation with $i^{\underline{\text{th}}}$ period.
Note that $|\,\Omega^i\,| = n_u^i$ (overhauling of units in periods is taken into account), there are single-period and multi-period non-LMCs, and that equality non-LMCs could be also included.

Stochastic medium-term power planning 2

The formulation of the stochastic medium-term minimum cost planning over an scenario tree would be:

$$\begin{aligned}
\underset{x_j^\nu}{\text{minimize}} \quad & \sum_{\nu\in\mathcal{N}} \pi_\nu \sum_{j=0}^{n_u} \widetilde{f}_j x_j^\nu \\
\text{subject to:} \quad & \sum_{j\in U} x_j^\nu \le \widehat{e}^{i(\nu)} - w^{i(\nu)}(U) \quad \forall U \subset \Omega^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& \sum_{j=0}^{n_u} x_j^\nu = \widehat{e}^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& A^{i(\nu)} x^\nu \ge r^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& \sum_{\nu\in\mathcal{H}(\lambda)} A^{\lambda,i(\nu)} x^\nu \ge r^\lambda \quad \forall \lambda \in \mathcal{L} \quad \text{(for each leaf !!)} \\
& x_j^{i(\nu)} \ge \underline{0} \qquad j = o, 1, \ldots, n_u \quad \forall \nu \in \mathcal{N}
\end{aligned}$$

where supraindex $^\nu$ means relation with $\nu^{\underline{\text{th}}}$ node, $\mathcal{L} := \{\nu \in \mathcal{N} | \, i(\nu) = n_i\}$ is the set of *leaf* (final period) nodes, and $\mathcal{H}(\lambda)$ the path from the root to node λ. With the notation employed there is no need of *non-anticipativity* constraints.

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Medium-Term Pure-Pool Electricity Markets

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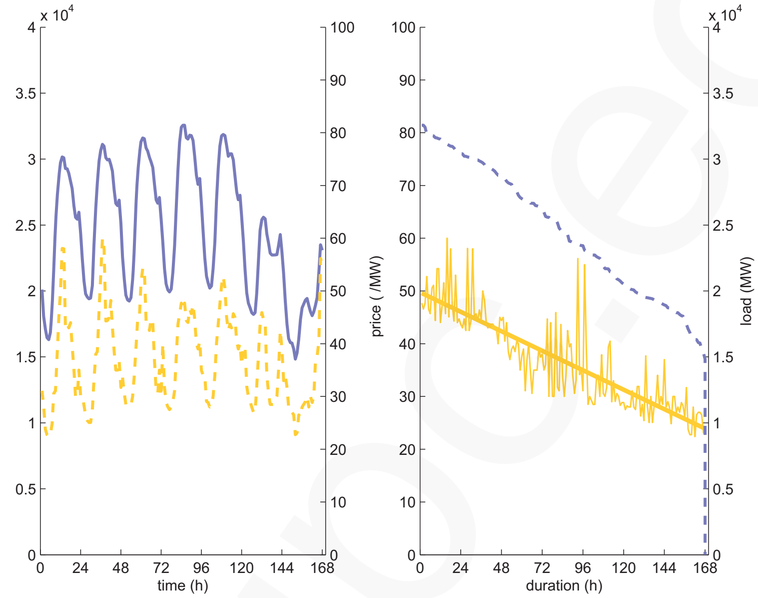
Medium-term market-price function

From the records of past market-price and load series it is possible to compute a market-price function for a given period. This function is to be used with expected generations that match the LDC of the period, so market prices should correspond in duration with the duration of loads, from peak to base load in the period.

The purpose of this function is to account for the fact that market price is not constant over the medium-term periods.



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Medium-term market-price function 2

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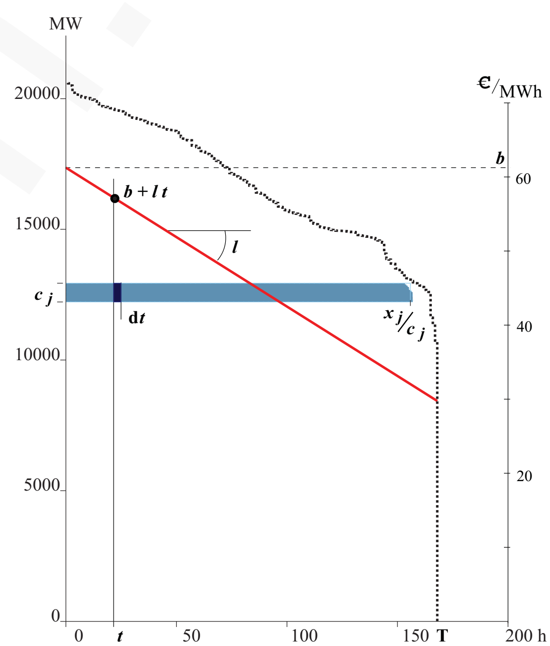

Medium-term market-price function w.r.t. the load duration for a time period and contribution of $j^{\underline{\text{th}}}$ unit.

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Pure-pool profit maximization function

The profit (revenue minus cost) of unit j in period i will be:

$$\int_0^{x_j^i/c_j} c_j\{b^i + l^i t - \widetilde{f}_j\}\mathrm{d}t = (b^i - \widetilde{f}_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2$$

and adding for all periods and units, and taking into account the external energy, we get the profit function to be maximized:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{(b^i - \widetilde{f}_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2\right\} - \widetilde{f}_0 x_0^i\right]$$

which is quadratic in the generated energies. Using the load balance equation we are led to the equivalent expression:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{(b^i - f_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2\right\} - \widetilde{f}_0 \widehat{e}^i\right]$$

with $f_j = \widetilde{f}_j - \widetilde{f}_0$

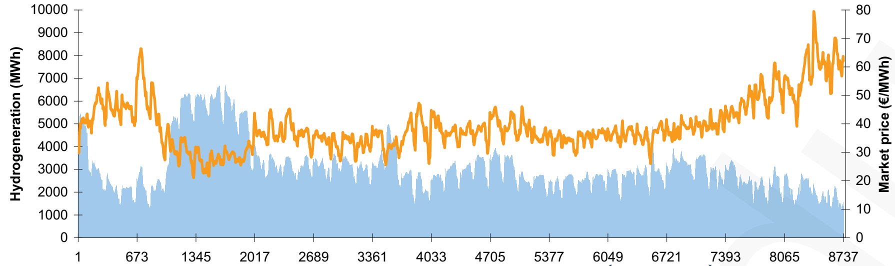
Generators' surplus (Cartel) problem

Given that $f_0\widehat{e}^i$ is a constant, the problem to be solved is:

$$\begin{aligned}
\underset{x_j^i}{\text{minimize}} \quad & \sum_i^{n_i}\sum_j^{n_u}\left\{(f_j - b^i)x_j^i - \frac{l^i}{2c_j}{x_j^i}^2\right\} \\
\text{subject to:} \quad & \sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i = 1,\dots,n_i \\
& A_{\ge}^i x^i \ge R_{\ge}^i \qquad i = 1,\dots,n_i \\
& \sum_i A_{\ge}^{0i} x^i \ge R_{\ge}^0 \\
& A_{=}^i x^i = R_{=}^i \qquad i = 1,\dots,n_i \\
& \sum_i A_{=}^{0i} x^i = R_{=}^0 \\
& x_j^i \ge \underline{0} \qquad j = 1,\dots,n_u, \quad i = 1,\dots,n_i
\end{aligned}$$

Given that $l^i < 0$, the quadratic of the objective function is positive definite, thus this problem has a unique global minimizer. Moreover, the quadratic of the objective function is diagonal. A multi-scenario version of this problem could be also formulated.

Endogenous modification of the market-price function



Weekly moving average of the market price (orange) and of hydro generation (blue area) during 2007 in the Spanish power pool.

The most obvious endogenous modification of the market-price function is that due to hydro generation. It can be clearly observed from historical records that when the hydro generation level increases, market prices tend to decrease.
Given that both the peak and the base power demand prices appear to be equally affected by the hydro generation level, a linear change in the basic coefficient b^i is introduced:

$$b^i = b_0^i - c_0^i \sum_{k \in H} x_k^i$$

where $H \subset \Omega$ is the set of hydro units and b_0^i and c_0^i are positive coefficients that are estimated from past market-price and hydro generation data.

Endogenous modification of the profit maximization function

Substituting in, integrating and simplyfying the profit maximization function we obtain:

$$\sum_{i}^{n_i} \left[\sum_{j}^{n_u} \{ (b_0^i - \widetilde{f}_j + \widetilde{f}_0) x_j^i - c_0^i \sum_{k \in H} x_k^i x_j^i + \frac{l^i}{2c_j} {x_j^i}^2 \} - \widetilde{f}_0 \widehat{e}^i \right],$$

which is still quadratic, but its matrix is no longer diagonal and it may be indefinite for values l^i and c_0^i found in practice.

Taking $\widetilde{f}_j - \widetilde{f}_0$ as f_j and removing the constants terms from the objective function we are left with the generators' surplus problem with endogenous influence of hydro.

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Pure-pool generators' surplus problem with endogenous influence of hydro

$$\begin{array}{lll}
\underset{x_j^i}{\text{minimize}} & \displaystyle\sum_i^{n_i}\sum_j^{n_u}\left\{(f_j - b^i)x_j^i + c_0^i \sum_{k\in H} x_k^i x_j^i - \frac{l^i}{2c_j} {x_j^i}^2\right\} & \\
\text{subject to:} & \displaystyle\sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i = 1,\ldots,n_i & \\
& A_{\ge}^i\, x^i \ge R_{\ge}^i \qquad i = 1,\ldots,n_i & \\
& \displaystyle\sum_i A_{\ge}^{0i}\, x^i \ge R_{\ge}^0 & \\
& A_{=}^i\, x^i = R_{=}^i \qquad i = 1,\ldots,n_i & \\
& \displaystyle\sum_i A_{=}^{0i}\, x^i = R_{=}^0 & \\
& x_j^i \ge \underline{0} \qquad j = 1,\ldots,n_u, \quad i = 1,\ldots,n_i &
\end{array}$$

in whose solution it can be observed that not all available hydro generation is spent in order to keep market prices, and profits, high.

Given that this situation does not occur in the Spanish pool, a non-LMC constraint is added that forces the reservoir systems to spend all received inflows within each year.

Nash-Cournot equilibrium in electricity markets 1

- A behavioural principle different from the generators' surplus maximization, which is monopolistic on the part of the generation companies, is the oligopolistic Nash equilibrium in a game with Cournot competition type, which means a higher degree of competition than the generators' surplus maximization.
- In a Nash-Cournot equilibrium we can assume either two (the SGC and the RoP), or more players (K generation companies, whose units are $\Omega_k \,|\, \Omega := \{\Omega_1, \Omega_2, \ldots, \Omega_K\}$).
- In the Cournot model of competition we assume that the decision (generation) of one player is conditioned by the decisions (generations) of the rest of the players and that the market price is a function of the overall decisions (total expected generation).
- In a Nash equilibrium no player can increase its revenue by unilaterally changing its decision (generation).
- It is not sure that a given pool behaves more like a Nash-Cournot equilibrium than like a monopolistic generators' surplus maximization.

The implementation of the NIRA algorithm to obtain the Nash-Cournot equilibrium

In case of using a scenario tree with nodes ν of the set $\mathcal{N}$, $\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widetilde{\mathbf{x}},\mathbf{x})$ would be equivalent to solving:

$$
\begin{aligned}
\underset{x_j^\nu}{\text{minimize}} \quad & \sum_{\nu\in\mathcal{N}} \pi_\nu \sum_{k=1}^{K} \sum_{j\in\Omega_k^{i(\nu)}} \Big\{ \big(f_j - b^{i(\nu)}\big) x_j^\nu + c_0^{i(\nu)} \Big[\sum_{l\in H_k} x_l^\nu x_j^\nu \\
& + \sum_{l\in H_m | m\neq k} \widetilde{x}_l^\nu x_j^\nu \Big] - \frac{l^{i(\nu)}}{2c_j} {x_j^\nu}^2 \Big\} \\
\text{subject to:} \quad & \sum_{j\in U} x_j^\nu \le \widehat{e}^{i(\nu)} - w^{i(\nu)}(U) \quad \forall U \subset \Omega^{i(\nu)} \quad \forall \nu\in\mathcal{N} \\
& A_{\ge}^{i(\nu)} x^\nu \ge r_{\ge}^{i(\nu)} \quad \forall \nu\in\mathcal{N} \qquad (1) \\
& \sum_{\nu\in\mathcal{H}(\lambda)} A_{\ge}^{\lambda,i(\nu)} x^\nu \ge r_{\ge}^{\lambda} \quad \forall \lambda\in\mathcal{L} \\
& A_{=}^{i(\nu)} x^\nu = r_{=}^{i(\nu)} \quad \forall \nu\in\mathcal{N} \\
& \sum_{\nu\in\mathcal{H}(\lambda)} A_{=}^{\lambda,i(\nu)} x^\nu = r_{=}^{\lambda} \quad \forall \lambda\in\mathcal{L} \\
& x_j^\nu \ge \underline{0} \quad j=1,\dots,n_u, \quad \forall \nu\in\mathcal{N}
\end{aligned}
$$

The classic NIRA algorithm to obtain the Nash-Cournot equilibrium

- $\widetilde{\mathbf{x}} \leftarrow \mathbf{x}_0\,, \quad u \leftarrow 0.7$
- **repeat**

  obtain $Z(\widetilde{\mathbf{x}}) = \mathbf{x}^*$ by solving $\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widetilde{\mathbf{x}},\mathbf{x})$ as in (1)

  compute $\Psi^* = \Psi(\widetilde{\mathbf{x}}, Z(\widetilde{\mathbf{x}})) = \sum_{k=1}^{K} \big(\phi_k(x_k^*|\widetilde{\mathbf{x}}) - \phi_k(\widetilde{\mathbf{x}})\big)$

  $\widetilde{\mathbf{x}} \leftarrow uZ(\widetilde{\mathbf{x}}) + (1-u)\widetilde{\mathbf{x}}$
- **until** $\Psi^* \le \epsilon$

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Renewable Energy Sources in Liberalized Electricity Markets

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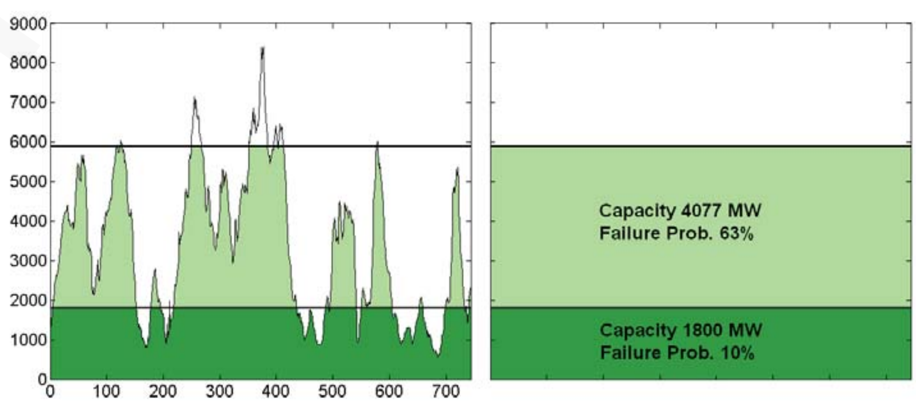
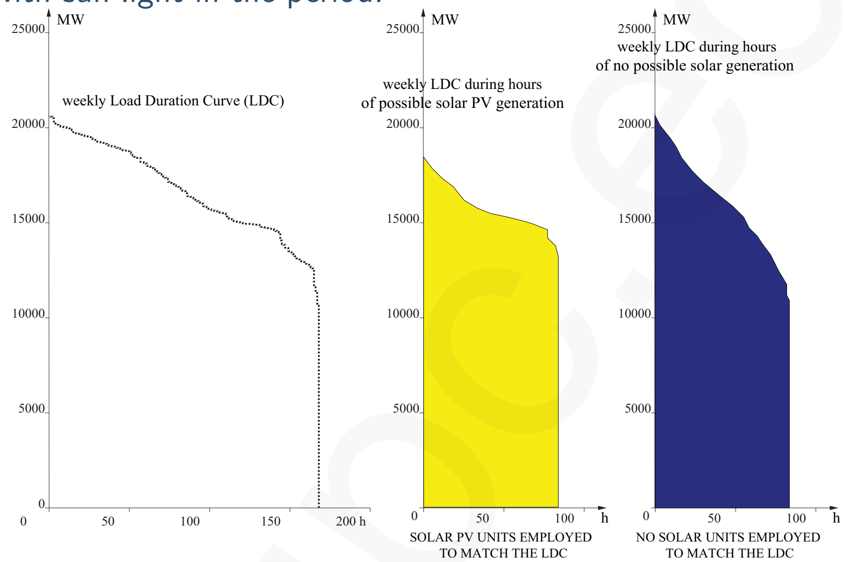
The representation of wind-power generation

- From the wind-power series corresponding to a given time period we deduce a two-unit model that represents its wind-power generation, with parameters suitable for being employed in the matching of the period LDC.
- Two pseudounits: the base unit and the crest unit. The spikes up to 2% of wind-power energy are neglegted.
- In the scenario generation the scenario tree nodes are based on base unit capacity (with fixed failure 10%). Crest units have fixed capacity and fixed failure probability for each period.



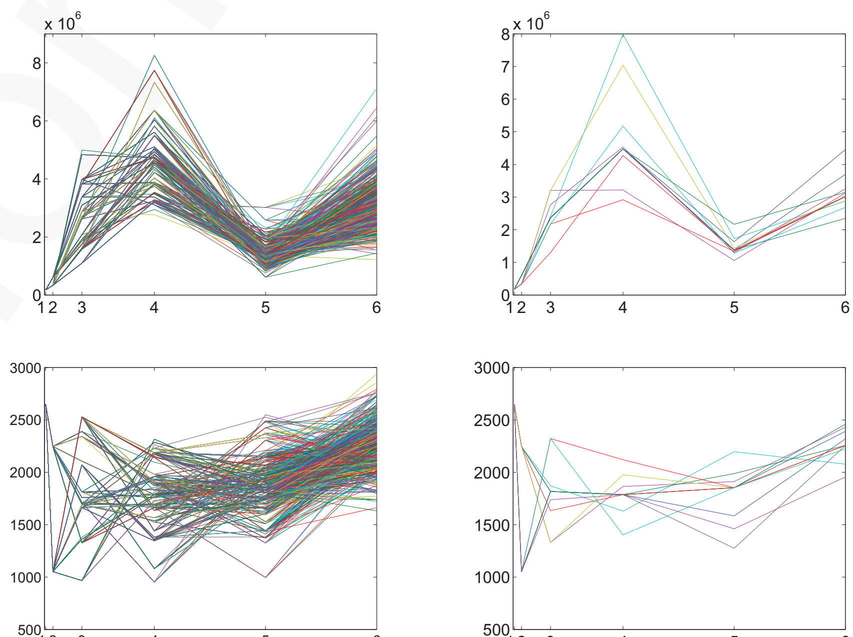
The representation of solar Photo-Voltaic (PV) generation

From the PV generation series corresponding to a given time period we deduce a two-unit model that represents its PV generation, using a two-unit model of base PV unit and crest PV unit as for wind-power generation.
An important difference with respect to wind power is that now each time period must be subdivided into two subperiods: one with no PV generation (no sun light hours), and another with it corresponding to hours with sun light in the period.



Scenario Generation and Scenario Reduction

- The scenario tree is created using a mixture of multidimensional vector auto regressive model and Montecarlo methods.
- We reduce the scenario tree to the desired number of scenarios using a backward algorithm



Hydro-inflow scenario tree (above) and wind-power scenario tree

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Value of change in stored water in leaves of scenario trees

It is assumed that a generation company (GenCo) will keep or it will spend a larger or lesser part of the water inflows depending on wheather the inflows are above or below its yearly average $\overline{W}_h$. The final water storage in reservoirs v^f_λ is fixed for each inflow scenario path λ.

$$v^f_\lambda = v^0 + 0,4\Big\{\big(\sum_{i\in\mathcal{H}(\lambda)} w^i_h\big) - \overline{W}_h\Big\}$$

(a 40% of inflow excess/shortage is kept/discharged)
The change, positive or negative, in stored water $v^f_\lambda - v^0$ for each path λ is valued at an average market-price value $\overline{\rho}$, so for each path there is an extra term in the objective function that corresponds to the value of the change in the stored water:

$$\overline{\rho} \times 0,4\Big\{\big(\sum_{i\in\mathcal{H}(\lambda)} w^i_h\big) - \overline{W}_h\Big\}$$

Medium-Term Mixed-System Electricity Markets

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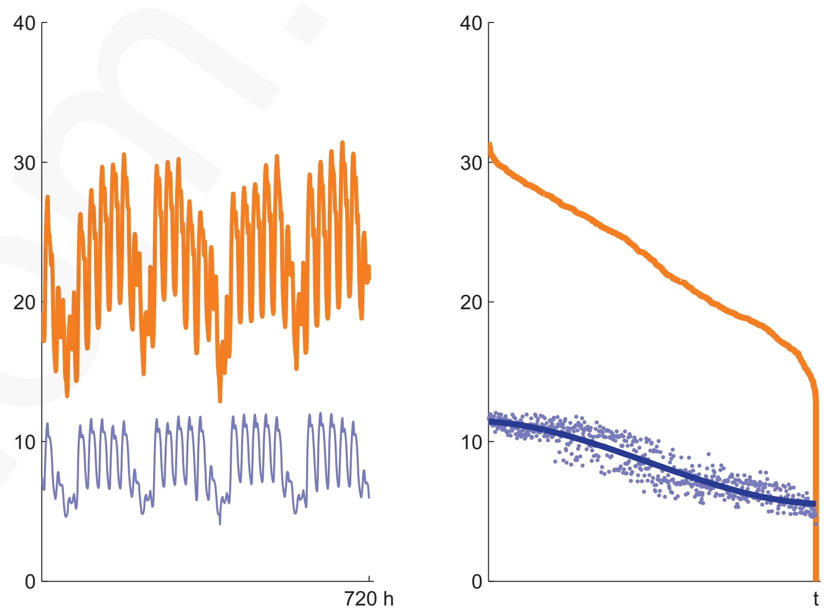
Bilateral contracts from the perspective of a SGC in medium-term power planning

- It is here assumed that, from current and past records of system and market load, acceptable predictions of load duration curves (LDCs) of system load and of BC load can be obtained, and that,
- through subtracting its own future BCs, the SGC is able to compute estimated future BC LDCs of the rest of participants (RoP) in the market, and that the SGC knows which are the technologies and capacities of the units of the RoP and has a sufficiently approximate knowledge of their generation cost and other parameters (such as the outage probability). Such information about loads and other generators' units is available at the Spanish Power Pool.
- In such conditions we are able to optimize the revenue from participating in the market while satisfying the BC load, but we must see how can we model that the SGC matches its own BC LDCs in successive periods while also contributing to match the market LDCs, and the RoP match their joint BC LDC while also contributing to match the market LDCs.
- The matching of an LDC will be modelled here through the linear inequality LMCs

System load and bilateral contracts load in a medium-term period



Series of the system demand and energy traded through bilateral contracts during June 2007 (left). LDC, bilateral data ordered according to the LDC and non-increasing fitted polynomial (right).

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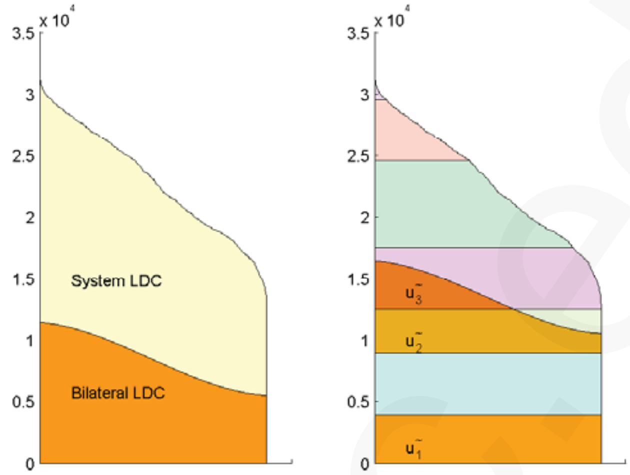
The time-share hypothesis in medium-term power planning with BCs

A time-share hypothesis is made to address the problem of each unit having the possibility of matching two different LDCs over a given period.



LDC of the system and part corresponding to the bilateral contracts LDC (shaded part, left), optimal load-matching with production for bilateral contracts (right). Zero outage probabilities assumed.

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Generation of SGC units for BCs and for Market

The revenue obtained from the market comes from the energy produced exceeding that devoted to the BCs. Let:
x_j: the total expected energy produced by unit j, and
$\widetilde{x}_j$: the expected energy devoted to match de BCs.
$x_j - \widetilde{x}_j$: the energy going to the market, which is paid at market price.
If we assume that the contribution of a unit has rectangular shape with height equal to its capacity, the market revenue for a unit is:

$$c_j \int_{\frac{\widetilde{x}_j}{c_j}}^{\frac{x_j}{c_j}} (b + lt)\, dt = b(x_j - \widetilde{x}_j) + \frac{1}{2}\frac{l}{c_j}(x_j^2 - \widetilde{x}_j^2)$$

which is a difference of convex functions.

Note that the part of the price function integrated starts after the expected time $\frac{\widetilde{x}_j}{c_j}$ devoted to generate for the BCs, where $\widetilde{x}_j \leq x_j$ stands for the energy generated by SGC unit j for the SGC BCs. The same type of revenue function applies to the units of the RoP using their generation $\breve{x}_k \leq x_k$ for the RoP BCs.
The costs incurred are:

- the generation costs for the whole generation x_j, and
- the cost of the external generation.

The medium-term power planning in a liberalized market with BCs

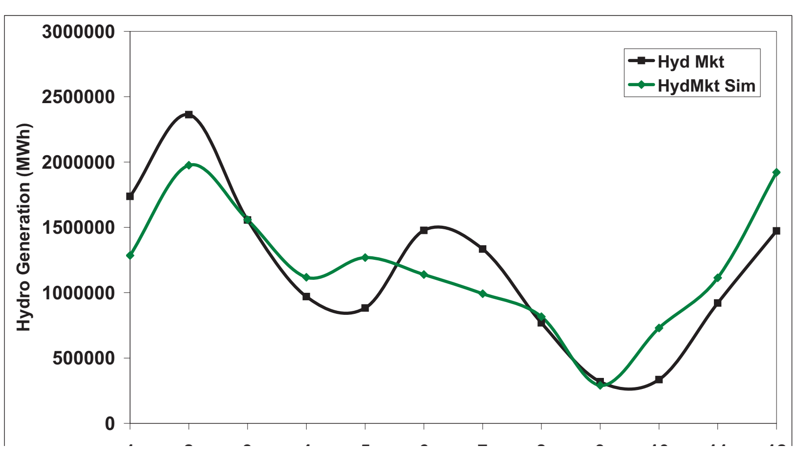
$$
\begin{array}{lll}
\underset{x,\widetilde{x},\breve{x}}{\text{maximize}} & \displaystyle\sum_{i=1}^{n_i}\Big[\sum_{j\in\widetilde{\Omega}^i}\{b^i(x_j^i-\widetilde{x}_j^i)+\frac{l^i}{2c_j}(x_j^{i\,2}-\widetilde{x}_j^{i\,2})\}+ & \\
 & \displaystyle\sum_{j\in\breve{\Omega}^i}\{b^i(x_j^i-\breve{x}_j^i)+\frac{l^i}{2c_j}(x_j^{i\,2}-\breve{x}_j^{i\,2})\}-\sum_{j\in\Omega}f_jx_j^i-f_0x_0^i\Big] & \\
\text{subject to} & \widetilde{x}_j^i\le x_j^i \quad j\in\widetilde{\Omega}^i \quad \forall i & \\
 & \breve{x}_j^i\le x_j^i \quad j\in\breve{\Omega}^i \quad \forall i & \\
 & \sum_{j\in\widetilde{\phi}^i}\widetilde{x}_j^i\le\widetilde{e}^i-w^i(\widetilde{\phi}^i) \quad \forall\,\widetilde{\phi}^i\subset\widetilde{\Omega}^i \quad \forall i & \\
 & \sum_{j\in\breve{\phi}^i}\breve{x}_j^i\le\breve{e}^i-w^i(\breve{\phi}^i) \quad \forall\,\breve{\phi}^i\subset\breve{\Omega}^i \quad \forall i & \\
 & \sum_{j\in\phi^i}x_j^i\le e^i-w^i(\phi^i) \quad \forall\,\phi^i\subseteq\Omega^i \quad \forall i & \\
 & \sum_{j\in\widetilde{\Omega}^i}\widetilde{x}_j^i=\widetilde{e}^i-w^i(\widetilde{\Omega}^i) \quad \forall i & \\
 & \sum_{j\in\breve{\Omega}^i}\breve{x}_j^i=\breve{e}^i-w^i(\breve{\Omega}^i) \quad \forall i & \\
 & \sum_{j\in\Omega^i}x_j^i+x_0^i=e^i \quad \forall i & \\
 & Cx\ge d & \\
 & 0\le\widetilde{x}_j^i\le\overline{\widetilde{x}}_j^i \quad j\in\widetilde{\Omega}^i \quad \forall i & \\
 & 0\le\breve{x}_j^i\le\overline{\breve{x}}_j^i \quad j\in\breve{\Omega}^i \quad \forall i & \\
 & 0\le x_j^i\le\overline{x}_j^i \quad j\in\Omega^i \quad \forall i\,, &
\end{array}
$$

where the o.f. is the difference of two convex functions (DC).

The hydro-to-market limit constraint

Both with cartel and with equilibrium behaviour GenCos would tend to conceal hydro from market by using it for BCs. This does not happen due to the regulations of the Energy Authorities.
A constraint is incorporated so that the amount of hydro generation bid in the market auction is similar to that observed in practice. The amount of recorded hydro traded in the market in each subperiod has been fit by a linear function of several parameters: the natural inflows w_h^i, the demand $\widehat{e}^i$, the stored hydro reserves $v^i=v^0+\sum_1^i(w_h^l-x_h^l)$ and tha average market price π^i

$$\sum_{h\in H}(x_h^i-\widetilde{x}_h^i)\ge\alpha w_h^i+\beta\hat{e}^i+\gamma v^i+\delta\pi^i \quad \forall i\in 1..n_i$$



Solution procedure for finding the equilibrium in mixed electricity markets

The expression of the utility of GenCo k has linear terms

$$b_0^i(x_j^i - \widetilde{x}_j^i) - f_j x_j^i + (x_j^i - \widetilde{x}_j^i)c_0^i \sum_{h\in\{H\setminus H_k\}} ({x'}_h^i - \widetilde{x'}_h^i)$$

(given that ${x'}_h^i$ and $\widetilde{x'}_h^i$ are here fixed), and quadratic nonconvex terms

$$\frac{l^i}{2c_j}({x_j^i}^2 - {\widetilde{x}_j^i}^2) + (x_j^i - \widetilde{x}_j^i)c_0^i \sum_{h\in H_k} (x_h^i - \widetilde{x}_h^i)$$

which, bearing in mind that c_0^i and l^i are negative, it can be decomposed as the difference of two concave (DC) functions:

$$\frac{l^i}{2c_j}{x_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i + \sum_{h\in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2$$

$$-\left[\frac{l^i}{2c_j}{\widetilde{x}_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i - \sum_{h\in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2\right]$$

Solution procedure for finding the equilibrium in mixed electricity markets

An alternative formulation, employed in global optimization for DC nonconvex problems is to maximize the concave part of the objective function, subject to a *reverse convex constraint* (RCC) that contains the convex part of the objective function:

$$\begin{aligned}
\underset{x_j^i,\widetilde{x}_j^i}{\text{maximize}} \quad & \sum_{i=1}^{n_i}\Bigg\{\sum_{j\in\Omega}\Bigg[b_0^i(x_j^i - \widetilde{x}_j^i) - f_j x_j^i + (x_j^i - \widetilde{x}_j^i)c_0^i \sum_{h\in\{H\setminus H_k\}} ({x'}_h^i - \widetilde{x'}_h^i) \\
& + \frac{l^i}{2c_j}{x_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i + \sum_{h\in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2\Bigg] - f_0 x_0^i\Bigg\} + z \\
\text{subject to} \quad & -\sum_{i=1}^{n_i}\Bigg\{\sum_{j\in\Omega}\Bigg[\frac{l^i}{2c_j}{\widetilde{x}_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i - \sum_{h\in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2\Bigg]\Bigg\} - z \geq 0 \\
& \text{rest of constraints: LMCs, nonLMCs, and bounds}
\end{aligned}$$

where the explicit constraint is the RCC.
Linearizing the RCC about previously obtained points and resolving the problem could be a strategy for approaching the global optimizer.

Conclusions

- A new model for a mixed market using a time-share hypothesis has been presented.
- The resulting problem has a non convex objective function.
- A Hydro-to-Market constraint is necessary.
- We found both the solution for the Cartel behaviour and Equilibrium behaviour using the Nikaido Isoda Relaxation Algorithm.
- The Equilibrium solution has profits lower than the Cartel solution, as expected.
- In the model presented, if not for the endogenous function due to hydro generation, we would not get an equilibrium solution.
- A new way to represent the wind-power generation with two pseudounits with given capacity and failure probability in each node of the scenario tree has been presented.
- No procedure that systematically obtains the best optimizer has been found yet for solving the DC mixed market power planning.

Thank you for your attention!

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